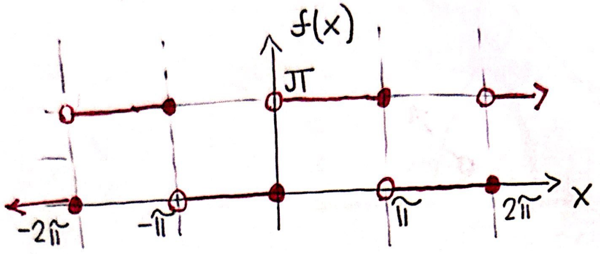
Matematika G2 - GY09

1. feladat

$$f(x)=\begin{cases} 0 & \text{ha } -\pi < x \le 0 \\ \pi & \text{ha } 0 < x \le \pi \end{cases} \qquad f(x)=f(x+2k\pi)$$



bonyolultabb mo.:

$$a_0=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx=\frac{1}{2\pi}\int_{-\pi}^{0} 0\,dx+\frac{1}{2\pi}\int_{0}^{\pi}\pi\,dx=\frac{\pi}{2}$$

$$a_k=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx=\frac{1}{\pi}\int_{0}^{\pi}\pi\cos(kx)\,dx=\left.\frac{\sin(kx)}{k}\right|_0^{\pi}=0$$

$$b_k=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx=\frac{1}{\pi}\int_{0}^{\pi}\pi\sin(kx)\,dx=\left.\frac{-\cos kx}{k}\right|_0^{\pi}=$$

$$=\frac{1-\cos k\pi}{k}=\begin{cases} 0, & \text{ha } k \text{ páros} \\ 2/k, & \text{ha } k \text{ páratlan}\end{cases}$$

$$f(x)=\frac{\pi}{2}+\frac{2}{1}\sin x+\frac{2}{3}\sin 3x+\ldots+\frac{2}{2k+1}\sin\left[(2k+1)x\right]+\ldots$$

$$=\frac{\pi}{2}+\sum_{k=0}^{\infty}\frac{\sin((2k+1)x)}{2k+1}$$

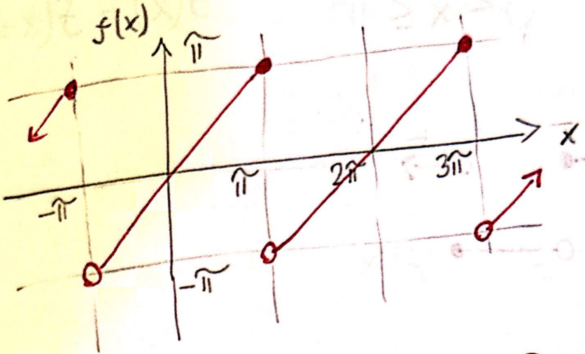
egyszerűbb mo.: $g(x)=f(x)-\pi/2 \Rightarrow g(x)$ páratlan

$a_0=a_k=0$

$$b_k=\int_{-\pi}^{0}\frac{-\sin(kx)}{2}\,dx+\int_{0}^{\pi}\frac{\sin(kx)}{2}\,dx=\int_{0}^{\pi}\sin(kx)\,dx=\ldots$$

2. feladat

$f(x) = x$, ha $x \in (-\pi; \pi]$ $\quad f(x) = f(x + 2k\pi)$



$f(x)$ páratlan $\Rightarrow a_0 = a_k = 0$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin kx \, dx = \frac{-x\cos kx}{\pi k}\Bigg|_{-\pi}^{\pi} + \cancel{\frac{1}{k\pi}\int_{-\pi}^{\pi} \cos kx \, dx}$$

$= 0$

$f = x \quad g = -\cos kx / k$

$f' = 1 \quad g' = \sin kx$

$$= \frac{1}{k\pi}\left[-\pi\cos k\pi - \pi\cos(-k\pi)\right] = \frac{-2}{k}\cos k\pi =$$

$$= \begin{cases} -2/k, \text{ ha } k \text{ páros} \\ 2/k, \text{ ha } k \text{ páratlan} \end{cases}$$

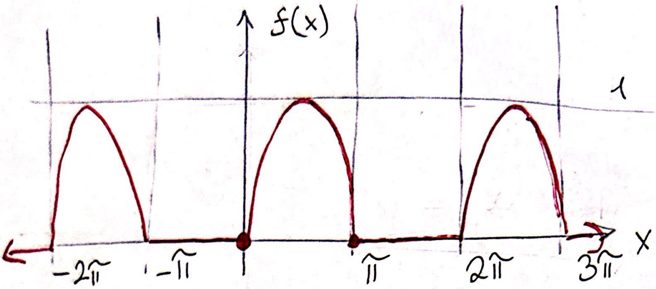
$$f(x) = \sum_{k=1}^{\infty} (-1)^{k+1} \cdot \frac{2}{k} \cdot \sin kx$$

$f(\pi) = ? \Rightarrow \sin k0 = 0 \Rightarrow f(\pi) = 0$

$$f(\pi) = \frac{1}{2}\lim_{x \to \pi^+} f(x) + \frac{1}{2}\lim_{x \to \pi^-} f(x)$$

3. feladat

$$f(x) = \begin{cases} \sin x & 0 \leq x < \pi \\ 0 & \pi \leq x < 2\pi \end{cases} \qquad f(x) = f(x + 2k\pi)$$



$$a_0 = \frac{1}{2\pi}\int_0^{\pi} \sin x\,dx = \frac{1}{2\pi}(-\cos x)\Big|_0^{\pi} = \frac{1}{2\pi}\Big(-(-1) - (-1)\Big) = \frac{1}{\pi}$$

$$a_1 = \frac{1}{\pi}\int_0^{\pi} \sin x \cos x\,dx = \frac{1}{2\pi}\int_0^{\pi} \sin 2x\,dx = 0$$

$$a_{k>1} = \frac{1}{\pi}\int_0^{\pi} \sin x \cos kx\,dx = \frac{1}{2\pi}\int_0^{\pi} \sin[(k+1)x] - \sin[(k-1)x]\,dx$$

$$= \frac{1}{2\pi}\left[\frac{\cos[(k-1)x]}{k-1} - \frac{\cos[(k+1)x]}{k+1}\right]_0^{\pi} =$$

$$= \frac{1}{2\pi}\left[\frac{(-1)^{k-1}}{k-1} - \frac{(-1)^{k+1}}{k+1} - \frac{1}{k-1} + \frac{1}{k+1}\right] = \begin{cases} 0, \text{ ha } k \text{ páratlan} \\ \Delta, \text{ ha } k \text{ páros} \end{cases}$$

$$\Delta = \frac{-2}{2\pi(k-1)} + \frac{-2}{2\pi(k+1)} = \frac{-(k-1)-(k+1)}{\pi(k^2-1)} = \frac{-2k}{\pi(k^2-1)}$$

$$b_1 = \frac{1}{\pi}\int_0^{\pi} \sin^2 x\,dx = \frac{1}{2}$$

$$b_{k>1} = \frac{1}{2\pi}\int_0^{\pi} \cos[(n-1)x] - \cos[(n+1)x]\,dx = 0$$

4. feladat

$f(x) = \sin^4 x$

$$\sin^4 x = \left(\frac{1-\cos 2x}{2}\right)^2 = \frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x =$$

$$= \dots + \frac{1+\cos 4x}{8} = \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$

$a_0 = 3/8 \quad a_2 = -1/2 \quad a_3 = 1/8$

$g(x) = \cos 3x \sin^2 x$

$$g(x) = \cos 3x \cdot \frac{1-\cos 2x}{2} =$$

$$= \frac{\cos 3x}{2} - \frac{\cos 3x \cdot \cos 2x}{2}$$

$$= \frac{\cos 3x}{2} - \frac{\cos 5x}{4} - \frac{\cos x}{4}$$

$\Rightarrow a_1 = -1/4 \quad a_3 = 1/2 \quad a_5 = -1/4$

Másik mo.: $\sin x = -i\sinh(ix)$ és $\cos x = \cosh(ix)$

$$g(x) = \frac{e^{3ix} + e^{-3ix}}{2} \cdot \left(\frac{e^{2ix} - e^{-2ix}}{2i}\right)^2 = \dots$$

5. feladat

$f(x) = x^2 \quad x \in (-1;1] \quad f(x+2) = f(x) \quad 2p = 2$

$$a_0 = \frac{1}{2}\int_{-1}^{1} x^2 dx = \int_0^1 x^2 dx = 1/3$$

$$a_k = \frac{1}{1}\int_{-1}^{1} x^2 \cos k\pi x \, dx = 2\int_0^1 x^2 \cos k\pi x \, dx$$

	D	I	
+	x^2	$\cos k\pi x$	
-	$2x$	$\frac{\sin k\pi x}{k\pi}$	~ 0
+	2	$-\frac{\cos k\pi x}{k^2\pi^2}$	
	0	$-\frac{\sin k\pi x}{k^3\pi^3}$	~ 0

$$= \left.\frac{+4\cos k\pi x}{k^2\pi^2}\right|_0^1 = (-1)^k \frac{4}{k^2\pi^2}$$

$b_k \equiv 0$