Matematika G2 - GY8

1. feladat

$p(x) = (1+x)^3 \qquad x_0 = 0 \qquad p(0) = 1$

$p'(x) = 3(1+x)^2 \qquad p'(0) = 3$

$p''(x) = 6(1+x) \qquad p''(0) = 6$

$p'''(x) = 6 \qquad p'''(0) = 6$

$$p(x) = 1 + \frac{3}{1!}x + \frac{6}{2!}x^2 + \frac{6}{3!}x^3 = 1 + 3x + 3x^2 + x^3 = (1+x)^3$$

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2. feladat

$f(x) = (1-x)^3 \qquad x_0 = 1 \qquad f(1) = 0$

$f'(x) = -3(1-x)^2 \qquad f'(1) = 0$

$f''(x) = 6(1-x) \qquad f''(1) = 0$

$f'''(x) = -6 \qquad f'''(1) = -6$

$$f(x) = \frac{-6}{3!}(x-1)^3 = -(x-1)^3 = (1-x)^3$$

$$\frac{1}{r} = \lim_{n \to \infty} \sqrt[n]{a_n} = 0 \Rightarrow r = +\infty \qquad K = \mathbb{R}$$

$g(x) = e^x \qquad x_0 = 1$

$g^{(n)}(x) = e^x \qquad g^{(n)}(1) = e$

$$g(x) = \sum_{k=0}^{\infty} \frac{e}{k!}(x-1)^k$$

$$\frac{1}{r} = \lim_{n \to \infty} \frac{e}{(n+1)!} \frac{n!}{e} = \lim_{n \to \infty} \frac{1}{n+1} = 0 \Rightarrow r = +\infty$$

$K = \mathbb{R}$

$h(x) = \ln x \qquad x_0 = 1 \qquad h(1) = 0$

$h'(x) = 1/x \qquad h'(1) = 1$

$h''(x) = -1/x^2 \qquad h''(1) = -1$

$h'''(x) = 2/x^3 \qquad h'''(1) = 2$

$\vdots \qquad \vdots$

$$h^{(n)} = \frac{(-1)^{n+1}(n-1)!}{x^n} \qquad h^{(n)} = \frac{(-1)^{n+1}(n-1)!}{1}$$

$$h(x) = \frac{(-1)^{n+1}}{n}(x-1)^n$$

$$\frac{1}{r} = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{n+1}{n} = 1 \Rightarrow r = 1$$

$$K = (0;2]$$

3. feladat

$f(x) = \cos 5x$

$$\cos(x) = \sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{(2k)!} \qquad r = \infty$$

$$\cos(5x) = \sum_{k=0}^{\infty}(-1)^k\frac{(5x)^{2k}}{(2k)!} \qquad r = \infty$$

$g(x) = \sin\sqrt{x}$

$$\sin x = \sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{(2k+1)!} \qquad r = \infty$$

$$\sin\sqrt{x} = \sum_{k=0}^{\infty}(-1)^k\frac{\sqrt{x}^{2k+1}}{(2k+1)!} \qquad r = \infty$$

$h(x) = \sin^2 x$

a, $\sin^2 x = \left[\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}\right]^2 = \sum_{k=0}^{\infty} \frac{x^{4k+2}}{(2k+1)!^2}$

b, $\sin^2 x = \frac{1-\cos 2x}{2} = \frac{1}{2} - \frac{1}{2}\cos 2x = \frac{1}{2} - \frac{1}{2} \sum (-1)^k \frac{(2x)^{2k}}{(2k)!}$

$i(x) = \sqrt[3]{\exp(-x^2)}$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \Rightarrow e^{-x^2} = \sum_{k=0}^{\infty} \frac{(-x^2)^k}{k!} \Rightarrow$$

$$\Rightarrow \sqrt[3]{e^{-x^2}} = \sqrt[3]{\sum \ldots}$$

4. feladat

$$f(x) = \frac{x+1}{x+3} \qquad x_0 = -2$$

$$\frac{x+1}{x+3} = \frac{x+3-2}{x+3} = 1 + \frac{-2}{x+3} = 1 + \frac{-2}{1+(x+2)}$$

$$\sum_{n=0}^{\infty} a_0 q^n = \frac{a_0}{1-q}$$

$a_0 = -2 \quad q = -(x+2)$

$$f(x) = 1 + \sum_{k=0}^{\infty} -2 \cdot (-1)^k (x+2)^k$$

$$= 1 + \sum_{k=0}^{\infty} 2(-1)^{k+1} (x+2)^k$$

$$= -1 + \sum_{k=1}^{\infty} 2(-1)^{k+1} (x+2)^k$$

$$g(x) = \frac{x+1}{x+3} \qquad x_0 = 1$$

$$\frac{x+1}{x+3} = 1 + \frac{-2}{x+3} = 1 + \frac{-2}{(x+1)+2} = 1 + \frac{-1}{\frac{x+1}{2}+1}$$

$$q = -\frac{x+1}{2} \quad a_0 = -1$$

$$\frac{x+1}{x+3} = 1 + \sum_{n=0}^{\infty} (-1)(-1)^n \left(\frac{x+1}{2}\right)^n = 1 + \sum_{n=0}^{\infty} (-1)^{n+1} \frac{(x+1)^n}{2^n}$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x+1)^n}{2^n}$$

5. feladat

$$f(x) = \frac{1}{x^2-3x+2} \qquad x_0 = -2$$

$$f(x) = \frac{1}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1} = \frac{1}{x-2} + \frac{-1}{x-1}$$

$$A = \frac{1}{x-2}\Big|_1 = -1 \qquad B = \frac{1}{x-1}\Big|_2 = 1$$

$$\frac{1}{x-1} = \frac{-1}{x+2-3} = \frac{-1}{3-(x+2)} = \frac{-1/3}{1-\frac{x+2}{3}} = \sum_{n=0}^{\infty} -\frac{1}{3} \frac{(x+2)^2}{3^n} \qquad R_1 = 3$$

$$\frac{-1}{x-2} = \frac{-1}{x+2-4} = \frac{1}{4-(x+2)} = \frac{1/4}{1-\frac{x+2}{4}} = \sum_{n=0}^{\infty} \frac{1}{4} \frac{(x+2)^2}{4^n} \qquad R_1 = 4$$

$$R = \min\{R_1; R_2\} = 3 \qquad K = (-5;1)$$

6. Seradat

$$f(x) = \frac{x}{1+x^2}$$

$$\ln(1+x^2)' = \frac{2x}{1+x^2} \Rightarrow f(x) = \frac{1}{2}\left(\ln(1+x^2)\right)'$$

$$\ln(1+x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{k+1}$$

$$\ln(1+x^2) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+2}}{k+1}$$

$$\ln(1+x^2)' = \sum_{k=0}^{\infty} 2(-1)^k x^{2k+1}$$

$$f(x) = \sum_{k=0}^{\infty} (-1)^k x^{2k+1}$$

$$g(x) = \arctan x$$

$$(\arctan x)' = \frac{1}{1+x^2} = \sum_{k=0}^{\infty} (-1)^k x^{2k} \qquad R = 1$$

$$\arctan x = \sum_{k=0}^{\infty} \int_0^x (-1)^k t^{2k} dt = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1}$$

$$h(x) = \frac{1}{\sqrt{2+x^2}} = (2+x^2)^{-1/2} = \frac{1}{\sqrt{2}}\left(1+\frac{x^2}{2}\right)^{-1/2}$$

$$(1+x)^{\alpha} = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k$$

$$h(x) = \sum_{k=0}^{\infty} \binom{-1/2}{k} \left(\frac{x^2}{2}\right)^k$$

7. feladat

$$\sum_{n=0}^{\infty} \frac{2n+1}{n!} x^{2n}$$

$$(2n+1)x^{2n} = (x^{2n+1})'$$

$$\sum_{n=0}^{\infty} \int_0^x \frac{2n+1}{n!} x^{2n}\,dx = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!} = x\sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = xe^{x^2}$$

$$f(x) = (xe^{x^2})' = e^{x^2}(1+2x)$$