Matematika G2 - GY4

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1. feladat

$$\varphi: \mathbb{R}^2 \to \mathbb{R}^2; \quad \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+y \\ 5xy \end{bmatrix}$$

Ellenőrzés összeadásra és skalárral való szorzásra: ! $v_1(x_1;y_1)$ $v_2(x_2;y_2)$

$$\varphi(\underline{v}_1+\underline{v}_2) = \begin{bmatrix} x_1+y_1+x_2+y_2 \\ 5(x_1+x_2)(y_1+y_2) \end{bmatrix} = \begin{bmatrix} x_1+x_2+y_1+y_2 \\ 5x_1y_1+5x_1y_2+5x_2y_1+5x_2y_2 \end{bmatrix}$$

$$\not\parallel$$

$$\varphi(\underline{v}_1)+\varphi(\underline{v}_2) = \begin{bmatrix} x_1+y_1 \\ 5x_1y_1 \end{bmatrix} + \begin{bmatrix} x_2+y_2 \\ 5x_2y_2 \end{bmatrix} = \begin{bmatrix} x_1+x_2+y_1+y_2 \\ 5x_1y_1+5y_1y_2 \end{bmatrix}$$

$\Downarrow$

nem lineáris

$$\psi: \mathbb{R}^2 \to \mathbb{R}^3; \quad \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x \\ y \\ x+y \end{bmatrix}$$

$$\psi(\underline{v}_1+\underline{v}_2) = \begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ x_1+x_2+y_1+y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \\ x_1+y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ x_2+y_2 \end{bmatrix} = \psi(\underline{v}_1)+\psi(\underline{v}_2) \checkmark$$

$$\psi(\lambda\underline{v}_1) = \begin{bmatrix} \lambda x_1 \\ \lambda y_1 \\ \lambda(x_1+y_1) \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ y_1 \\ x_1+y_1 \end{bmatrix} = \lambda\psi(\underline{v}_1) \checkmark$$

$\Downarrow$

lineáris $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

2. feladat

$P(5;-4;-1)$ $\underline{a}_1(2;1;0)$ $\underline{a}_2(0;2;1)$ $\underline{a}_3(1;0;2)$

$$\left.\begin{aligned} \alpha\underline{a}_1+\beta\underline{a}_2+\gamma\underline{a}_3 &= 5\hat{i}-4\hat{j}-\hat{k} \\ \alpha(2\hat{i}+\hat{j})+\beta(2\hat{j}+\hat{k})+\gamma(2\hat{k}+\hat{i}) \end{aligned}\right\} \quad \begin{aligned} 2\alpha+\gamma &= +5 \\ 2\beta+\alpha &= -4 \\ 2\gamma+\beta &= -1 \end{aligned}$$

$\Downarrow$ LER

$\alpha = 2$

$\beta = -3$

$\gamma = 1$

3. feladat

$\{\hat{i};\hat{j};\hat{k}\} \to \{\hat{z}_1;\hat{z}_2;\hat{z}_3\}$

$$x\hat{i}+y\hat{j}+z\hat{k} = \xi\hat{z}_1+\eta\hat{z}_2+\zeta\hat{z}_3 = \begin{bmatrix}z_{11}\\z_{12}\\z_{13}\end{bmatrix}\xi + \begin{bmatrix}z_{21}\\z_{22}\\z_{23}\end{bmatrix}\eta + \begin{bmatrix}z_{31}\\z_{32}\\z_{33}\end{bmatrix}\zeta$$

$$\left.\begin{aligned} x &= z_{11}\xi + z_{21}\eta + z_{31}\zeta \\ y &= z_{12}\xi + z_{22}\eta + z_{32}\zeta \\ z &= z_{13}\xi + z_{23}\eta + z_{33}\zeta \end{aligned}\right\} \Rightarrow$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} + \underbrace{\begin{bmatrix}z_{11} & z_{21} & z_{31}\\ z_{12} & z_{22} & z_{32}\\ z_{13} & z_{23} & z_{33}\end{bmatrix}}_{\underline{\underline{T}}}\begin{bmatrix}\xi\\\eta\\\zeta\end{bmatrix}$$

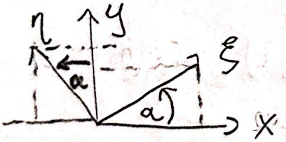
$\underline{x} = \underline{\underline{T}}\,\underline{x}'$ $\quad \underline{x}' = \underline{\underline{T}}^{-1}\underline{x}$ $\Leftarrow$

4. feladat

$$\underline{\underline{T}} = \begin{bmatrix}2 & 0 & 1\\ 1 & 2 & 0\\ 0 & 1 & 2\end{bmatrix} \qquad \underline{\underline{T}}^{-1} = \begin{bmatrix}4 & 1 & -2\\ -2 & 4 & 1\\ 1 & -2 & 4\end{bmatrix}\cdot\frac{1}{9}$$

$$\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix} = \underline{\underline{T}}^{-1}\begin{bmatrix}5\\-4\\-1\end{bmatrix} = \frac{1}{9}\begin{bmatrix}4 & 1 & -2\\ -2 & 4 & 1\\ 1 & -2 & 4\end{bmatrix}\begin{bmatrix}5\\-4\\-1\end{bmatrix} = \frac{1}{9}\begin{bmatrix}5\cdot4-4\cdot1-1(-2)\\ 5(-2)-4\cdot4-1\cdot1\\ 5\cdot1-4(-2)-1\cdot4\end{bmatrix} = \begin{bmatrix}2\\-3\\1\end{bmatrix}$$

5. feladat



$$\underline{b}_1 = \begin{bmatrix}\cos\alpha\\ +\sin\alpha\end{bmatrix} \qquad \underline{b}_2 = \begin{bmatrix}-\sin\alpha\\ \cos\alpha\end{bmatrix}$$

$$\underline{\underline{T}} = \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$(\det\underline{\underline{T}} = \sin^2\alpha + \cos^2\alpha = 1 \to$ ortogonális$)$

6. feladat

$\hat{i} \mapsto (2;1;3)$ $\hat{j} \mapsto (5;5;5)$ $\hat{k} \mapsto (0;0;1)$ $P(1;1;1)$

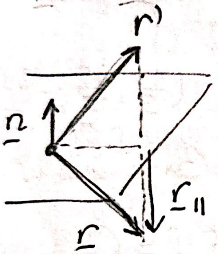
$$\underline{\underline{T}} = \begin{bmatrix} 2 & 5 & 0 \\ 1 & 5 & 0 \\ 3 & 5 & -1 \end{bmatrix} \qquad P' = \underline{\underline{T}}\,P = \begin{bmatrix} 2+5 \\ 1+5 \\ 3+5-1 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ 7 \end{bmatrix}$$

7. feladat

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ x+y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

8. feladat

$\underline{n}(a;b;c) \sim \|\underline{n}\| = 1$



$$\underline{r}' = \underline{r} - 2\underline{r}_{\parallel} = \underline{r} - 2(\underline{r}\cdot\underline{n})\underline{n} =$$
$$= \underline{r} - 2\underline{n}(\underline{n}\cdot\underline{r})$$

$$\underline{r}' = \underline{r} - 2\underline{n}\,\underline{n}^T\underline{r} = (\underline{\underline{E}} - 2\underline{n}\,\underline{n}^T)\,\underline{r}$$

$$\underline{n}\,\underline{n}^T = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$\underline{\underline{T}} = \underline{\underline{E}} - 2\underline{n}\,\underline{n}^T = \begin{bmatrix} 1-2a^2 & -2ab & -2ac \\ -2ab & 1-2b^2 & -2bc \\ -2ac & -2bc & 1-2c^2 \end{bmatrix}$$

$\det \underline{\underline{T}} = 1$

↓

ortogonális

orientációváltó

9. feladat

$\underline{f}_1(2;1)$ $\underline{f}_2(1;1)$ $(x;y) \mapsto (4x-2y;\, x+y)$

$$\underline{\underline{A}} = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} \qquad \underline{\underline{T}} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \qquad \underline{\underline{T}}^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\hat{\underline{\underline{A}}} = \underline{\underline{T}}^{-1}\underline{\underline{A}}\,\underline{\underline{T}} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\left.\begin{array}{l} \underline{f}_1 \text{ irányába } 3x \\ \underline{f}_2 \text{ irányába } 2x \end{array}\right\} \text{nyújtás}$$

10. feladat

$(\underline{\underline{A}}+\underline{\underline{B}})\underline{r} \to \underline{\underline{A}}\underline{r}+\underline{\underline{B}}\underline{r} \to$ külön külön lehet hatlatni

$\underline{\underline{A}}\underline{\underline{B}}\underline{r} \to \underline{\underline{A}}(\underline{\underline{B}}\underline{r}) \to$ először $\underline{\underline{B}}$ aztán $\underline{\underline{A}}$

$\underline{\underline{A}}^2\underline{r} \to 2\times \underline{\underline{A}}$

$\underline{\underline{A}}^{-1}\underline{r} \to$ inverz trafó

11. feladat

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -2 \\ 3 & -1 & 4 \end{bmatrix}$$

a, $P(2;0;1) \overset{?}{\in} \ker\varphi$

$\underline{\underline{A}}P = (5;-4;10) \neq \underline{0} \Rightarrow \notin$

b, $Q'(1;4;0)$ ősképe : $\underline{\underline{A}}Q = Q'$ $\quad Q = \underline{\underline{A}}^{-1}Q'$ $\left(\frac{46}{3};\frac{22}{3};-\frac{29}{3}\right)$

c, $\dim\varphi = ?$ $\quad \det\underline{\underline{A}} = -3 \neq 0 \Rightarrow \dim\varphi = 3$

d, $\operatorname{def}\varphi = ?$ $\quad \dim\varphi + \operatorname{def}\varphi = \dim V_1 \Rightarrow \operatorname{def}\varphi = 0$

12. feladat

$\operatorname{def}\varphi = ?$

a, x-tengelyre vetítés : $\dim\varphi = 1$ $\quad \operatorname{def}\varphi = 2$

b, yz-síkra vetítés : $\dim\varphi = 2$ $\quad \operatorname{def}\varphi = 1$

13. feladat

$R_z(\alpha)$, xy síkra tükröz, x ×2 z×3 nyújtás

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} 2c & -3s & 0 \\ s & c & 0 \\ 0 & 0 & -3 \end{bmatrix} ; \quad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 2c \\ s \\ -3 \end{bmatrix}$$

14. feladat

$\cos 45^\circ = 1/\sqrt{2}$ $\quad R_z = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\sin 45^\circ = 1/\sqrt{2}$

$\sin -45^\circ = -1/\sqrt{2}$

1. $R_z(-45^\circ)$
2. Tükröz x-tengelyre
3. $R_z(+45^\circ)$

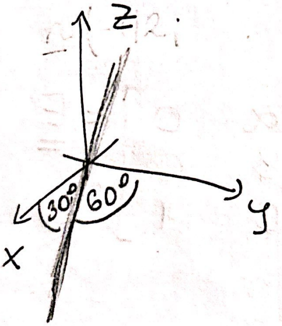
$$\underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}}_{3.} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{2.} \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}}_{1.} =$$

$$\frac{1}{2}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

15. feladat

$-\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 0 \qquad z = 0$

$\underline{n} = \begin{bmatrix} -1/2 \\ \sqrt{3}/2 \end{bmatrix} \qquad \underline{v} = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} \cos 30^\circ \\ \sin 30^\circ \end{bmatrix}$



① $\underline{\underline{R}}_z(60^\circ) = \begin{bmatrix} 1/2 & \sqrt{3}/2 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

② $\underline{\underline{R}}_y(+90^\circ) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

③ $\underline{\underline{R}}_z(-60^\circ) = \underline{\underline{R}}_z^T(60^\circ)$

$$\underline{\underline{T}} = \underline{\underline{R}}_z(-60^\circ)\,\underline{\underline{R}}_y(90^\circ)\,\underline{\underline{R}}_z(60^\circ) = \begin{bmatrix} 3/4 & \sqrt{3}/4 & 1/2 \\ \sqrt{3}/4 & 1/4 & -\sqrt{3}/2 \\ -1/2 & \sqrt{3}/2 & 0 \end{bmatrix}$$

$$\underline{\underline{T}} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \underline{\underline{T}}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\underline{\underline{A}}} = \underline{\underline{T}}^{-1} \underline{\underline{A}}\, \underline{\underline{T}} = \begin{bmatrix} 3/4 - \sqrt{3}/4 & 1/2 & 1+\sqrt{3}/2 \\ 1/2+\sqrt{3}/4 & 3/4-\sqrt{3}/4 & 3/4 - 3\sqrt{3}/4 \\ -1/2 & -1/2+\sqrt{3}/2 & -1/2+\sqrt{3}/2 \end{bmatrix}$$

16. feladat

$$\underline{\underline{A}} = \begin{bmatrix} \alpha & \beta & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \det \underline{\underline{A}} = -\beta = 1$$

$$\beta = -1$$

$$\underline{\underline{A}}^T = \begin{bmatrix} \alpha & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{A}} \cdot \underline{\underline{A}}^T = \begin{bmatrix} \alpha & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2+1 & \alpha & 0 \\ \alpha & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \underline{\underline{E}}$$

$\alpha^2+1 = 1$ és $\alpha = 0 \Rightarrow \alpha = 0$