

1. feladat

 $m \times n \quad A \quad \text{rg } A \quad \text{rg}(A|b)$

$M_{4 \times 3}$	3	4
$M_{3 \times 3}$	2	2
$M_{6 \times 8}$	6	6
$M_{8 \times 6}$	6	6
$M_{3 \times 2}$	0	0

 $\sim$ nincs mo. ($\text{rg } A \neq \text{rg}(A|b)$)

 $\sim \infty$ mo $3-2=1$ paraméter

 $\sim \infty$ mo $8-6=2$ paraméter

 ~ 1 mo

 $\sim \infty$ mo $2-0=2$ paraméter

 $\underline{A}$ nullmátrix

2. feladat

$$\underline{A} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \\ 1 & 4 & 3 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} -S_1 \\ -S_1 \\ -S_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array} \right] -3S_2 - 3S_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array} \right] \begin{array}{l} \Leftrightarrow \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array} \right]$$

$$\underline{x} = \underline{A}^{-1} \underline{b} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 7 \cdot 3 - 3 \cdot 6 - 3 \cdot 9 \\ -3 + 9 \\ -3 + 6 \end{bmatrix} = \begin{bmatrix} -24 \\ 6 \\ 3 \end{bmatrix}$$

3. seradat

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} \quad \underline{\underline{B}} = \begin{bmatrix} 5 \\ 1 \end{bmatrix} \quad \underline{\underline{Ax}} = \underline{\underline{B}} + 2x$$

$$\underline{\underline{Ax}} - 2x = \underline{\underline{B}}$$

$$(\underline{\underline{A}} - 2\underline{\underline{E}})x = \underline{\underline{B}}$$

$$x = (\underline{\underline{A}} - 2\underline{\underline{E}})^{-1} \underline{\underline{B}}$$

$$\underline{\underline{A}} - 2\underline{\underline{E}} = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix} = \underline{\underline{A^*}}$$

$$\left. \begin{array}{l} \det \underline{\underline{A^*}} = -1 \cdot 1 - (-2) \cdot 2 = 3 \\ \text{adj } \underline{\underline{A^*}} = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \end{array} \right\} \underline{\underline{A^*}}^{-1} = \frac{1}{3} \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$$

$$x = \frac{1}{3} \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 5-2 \\ 10-1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 3 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

4. seradat

$$\underline{\underline{A}} = \begin{bmatrix} 2 & -3 & 1 \\ -3 & 4 & -2 \\ 5 & 0 & 4 \end{bmatrix}$$

$$\det \underline{\underline{A}} = 5 \begin{vmatrix} -3 & 1 \\ 4 & -2 \end{vmatrix} + 4 \begin{vmatrix} 2 & -3 \\ -3 & 4 \end{vmatrix} =$$

$$= 5 \cdot 2 + 4(-1) = 6$$

$$\underline{\underline{b}} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$$

$$\underline{\underline{A}}_1 = \begin{bmatrix} \boxed{0} & -3 & 1 \\ 1 & 4 & -2 \\ -3 & 0 & 4 \end{bmatrix}$$

$$\underline{\underline{A}}_2 = \begin{bmatrix} 2 & \boxed{0} & 1 \\ -3 & 1 & -2 \\ 5 & -3 & 4 \end{bmatrix}$$

$$\underline{\underline{A}}_3 = \begin{bmatrix} 2 & -3 & \boxed{0} \\ -3 & 4 & 1 \\ 5 & 0 & -3 \end{bmatrix}$$

$$\det \underline{\underline{A}}_1 = 6$$

$$\det \underline{\underline{A}}_2 = 0$$

$$\det \underline{\underline{A}}_3 = -12$$

$$x_1 = 6/6 = 1$$

$$x_2 = 0$$

$$x_3 = -12/6 = -2$$

$$\left. \begin{array}{l} \det \underline{\underline{A}}_1 = 6 \\ \det \underline{\underline{A}}_2 = 0 \\ \det \underline{\underline{A}}_3 = -12 \end{array} \right\} x = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

5. Seladat

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

$$\underline{\underline{b}} = \begin{bmatrix} 5 \\ 1 \\ 1 \\ -5 \end{bmatrix}$$

$$\underline{\underline{A}} \underline{\underline{x}} = \underline{\underline{b}}$$

$$x_1, x_2 = ?$$

matrixcalc :

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 2 & 3 & 1 \\ 3 & 2 & 1 & 2 & 1 \\ 4 & 3 & 2 & 1 & -5 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1/3 & 2/3 & -1 \\ 0 & 1 & 4/3 & 5/3 & 3 \\ 0 & 0 & 1 & 15/12 & 5/4 \\ 0 & 0 & -10/3 & 5/3 & -70 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -7 \\ 2 & -4 & 1 & 5 \\ 0 & 0 & 0 & -1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -7 \\ 0 & 8 & -2 & -2 \\ 0 & 2 & 0 & 8 \end{array} \right]$$

6. feladat

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 3 & -1 & 2 & 7 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 1 & 7 \end{array} \right] \begin{array}{l} -3S_1 \\ -S_1 \\ -2S_1 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -7 & 5 & 1 \\ 0 & -2 & 0 & -4 \\ 0 & -3 & 3 & 3 \end{array} \right] -3S_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -1 & 5 & 13 \\ 0 & -2 & 0 & -4 \\ 0 & -3 & 3 & 3 \end{array} \right] \cdot (-1) \sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 1 & -5 & -13 \\ 0 & -2 & 0 & -4 \\ 0 & -3 & 3 & 3 \end{array} \right] \begin{array}{l} -2S_2 \\ +2S_2 \\ +3S_2 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 9 & 28 \\ 0 & 1 & -5 & -13 \\ 0 & 0 & -10 & -30 \\ 0 & 0 & -12 & -36 \end{array} \right] \begin{array}{l} \\ \\ /-10 \\ /-12 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 9 & 28 \\ 0 & 1 & -5 & -13 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 3 \end{array} \right] \begin{array}{l} +8S_3 \\ +5S_3 \\ -S_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} x_1 = 1 \\ x_2 = 2 \\ x_3 = 3 \end{array}$$

7. feladat

$$\left[\begin{array}{cccc|c} 5 & -1 & 2 & 1 & 7 \\ 2 & 1 & 4 & -2 & 1 \\ 1 & -3 & -6 & 5 & 0 \end{array} \right] +2S_2 - S_1 \sim \left[\begin{array}{cccc|c} \cdot & \cdot & \cdot & \cdot & 7 \\ \cdot & \cdot & \cdot & \cdot & 1 \\ 0 & 0 & 0 & 0 & 9 \end{array} \right]$$

$\text{rg } \underline{A} < \text{rg } (\underline{A}|\underline{b}) \Rightarrow$ nincs mo.

8. feladat

$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ -2 & -2 & -6 & -8 & 0 & 10 \\ 2 & 2 & -3 & -1 & 4 & 17 \\ -1 & -1 & 1 & 0 & 4 & 9 \\ 3 & 3 & 6 & 9 & 8 & 15 \end{array} \right] \begin{array}{l} +2S_1 \\ -2S_1 \\ +S_1 \\ -3S_1 \end{array} \sim \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & -4 & -4 & 2 & 14 \\ 0 & 0 & -5 & -5 & 1 & 13 \\ 0 & 0 & 2 & 2 & 5 & 11 \\ 0 & 0 & 3 & 3 & 5 & 9 \end{array} \right] -S_3$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & -5 & -5 & 1 & 13 \\ 0 & 0 & 2 & 2 & 5 & 11 \\ 0 & 0 & 3 & 3 & 5 & 9 \end{array} \right] \begin{array}{l} +5S_2 \\ -2S_2 \\ -3S_2 \end{array} \sim \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 6 & 18 \\ 0 & 0 & 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & 0 & 2 & 6 \end{array} \right] /6$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

paraméterek: x_4 és x_5

$$(3) \rightarrow x_5 = 3$$

$$(2) \rightarrow x_3 + x_4 + x_5 = 1$$

$$x_3 = 1 - x_4 - x_5 = -2 - x_4$$

$$(1) \rightarrow x_1 + x_2 + x_3 + 2x_4 + x_5 = 2$$

$$x_1 + x_2 + 1 + x_4 = 2$$

$$x_1 = 1 - x_2 - x_4$$

9. feladat

$$\underline{A} = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \begin{matrix} -2S_1 \\ -3S_1 \\ -S_1 \end{matrix} \quad \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix} \begin{matrix} +2S_2 \\ -2S_2 \end{matrix} \quad \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\text{rg } \underline{A} = 2 \neq 3 \Rightarrow$ nem csak triviális mo!

1 paraméter: x_3

$$-7x_2 - x_3 = 0 \quad x_2 = -x_3/7$$

$$x_1 + 3x_2 + 2x_3 = 0 \quad x_1 = \frac{+3x_3}{7} - 2x_3 = \frac{-11x_3}{7}$$

10. feladat

$$\underline{A} = \begin{bmatrix} 1 & 1 & -4 \\ 4 & 7 & 5 \\ 3 & 5 & 2 \\ 2 & 9 & 6 \end{bmatrix} \begin{matrix} -4S_1 \\ -3S_1 \\ -2S_1 \end{matrix} \quad \begin{bmatrix} 1 & 1 & -4 \\ 0 & 3 & 21 \\ 0 & 2 & 14 \\ 0 & 7 & 14 \end{bmatrix} \begin{matrix} /3 \\ \updownarrow \end{matrix} \quad \begin{bmatrix} 1 & 1 & -4 \\ 0 & 1 & 7 \\ 0 & 7 & 14 \\ 0 & 2 & 14 \end{bmatrix} \begin{matrix} /7 \\ /12 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & -4 \\ 0 & 1 & 7 \\ 0 & 1 & 7 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -4 \\ 0 & 1 & 7 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{rg } \underline{A} = 3$$

$\Downarrow$

csak triviális

11. feladat

$$\underline{A} = \begin{bmatrix} 2 & 1 & -4 \\ 3 & 5 & -7 \\ 4 & -5 & 6 \end{bmatrix}$$

$$\det \underline{A} = 2 \cdot 5 \cdot 6 + 1(-7)4 + (-4)3(-5) - 4 \cdot 5(-4) - (-5)(7)2 - 6 \cdot 3 \cdot 1 \\ = 60 - 28 + 60 + 80 - 70 - 18 = 84$$

$$\Rightarrow \operatorname{rg} \underline{A} = 3 \Rightarrow \text{csak triviális mo.}$$

12. feladat

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 2 & 3 & -3 & -2 & 4 \\ -3 & -5 & 4 & 1 & C \end{array} \right] \begin{array}{l} -2S_1 \\ +3S_1 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & -1 & -1 & -4 & 0 \\ 0 & 1 & 1 & 4 & 6+C \end{array} \right] + S_2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 2 \\ 0 & -1 & -1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 6+C \end{array} \right]$$

Megoldható, ha $C = -6$ ($\operatorname{rg} \underline{A} = \operatorname{rg}(\underline{A}|\underline{b})$)

$n = 4$ és $\operatorname{rg} \underline{A} = 2 \Rightarrow 2$ paraméter

13. feladat

$$\operatorname{rg} \underline{A} \leq 4 \text{ és } n=5$$

$\Downarrow$

Létezik nem triviális mo.

14. feladat

$$\left[\begin{array}{ccc|c} 3 & 5 & -1 & 1 \\ 1 & u & 2 & 2 \\ 1 & 9 & -5 & v \end{array} \right]$$

$$\det \underline{A} = -15u + 10 - 9 + u - 54 + 25 =$$

$$= -14u - 28$$

$$\det \underline{A} = 0 \Leftrightarrow u = -2 \Rightarrow \operatorname{rg} \underline{A} = 2$$

$$\det \underline{A} \neq 0 \Leftrightarrow u \neq -2 \Rightarrow 1 \text{ db mo } (\operatorname{rg} \underline{A} = \operatorname{rg}(\underline{A}|\underline{b}))$$

Ha $u = -2$

$$\left[\begin{array}{ccc|c} 3 & 5 & -1 & 1 \\ 1 & -2 & 2 & 2 \\ 1 & 9 & -5 & v \end{array} \right] \begin{array}{l} -S_1/3 \\ -S_1/3 \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 5 & -1 & 1 \\ 0 & -\frac{11}{3} & \frac{7}{3} & \frac{5}{3} \\ 0 & \frac{22}{3} & -\frac{14}{3} & v - \frac{1}{3} \end{array} \right] \begin{array}{l} -S_1/3 \\ -S_1/3 \end{array}$$

$$(-2) \frac{5}{3} = v - \frac{1}{3}$$

$\Downarrow$

$$v = -3$$

$\Downarrow$

$$\operatorname{rg}(\underline{A}|\underline{b}) = 2$$

$\Downarrow$
 $\infty \text{ mo.}$

Ha $v \neq -2 \Rightarrow$ nincs mo