Matematika G2 - GY2

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1. feladat

$$\left.\begin{matrix} A(2;3;-4) \\ B(3;-1;-6) \\ C(-1;5;2) \\ D(2;1;4) \end{matrix}\right\}$$ Egy síkban vannak?

$$\left.\begin{matrix} \underline{a} = \overrightarrow{AB}(1;-4;-2) \\ \underline{b} = \overrightarrow{AC}(-3;2;6) \\ \underline{c} = \overrightarrow{AD}(0;-2;0) \end{matrix}\right\}$$ LIN FÜGGETLENEK-E?

$$\Updownarrow$$

$$\det(\underline{a}\ \underline{b}\ \underline{c}) \overset{?}{=} 0$$

1. Kifejtési tétel

$$\begin{vmatrix} 1 & -4 & 10 \\ -3 & 2 & 6 \\ 0 & -2 & 0 \end{vmatrix} = +0\begin{vmatrix} -4 & -2 \\ 2 & 6 \end{vmatrix} - (-2)\begin{vmatrix} 1 & -2 \\ -3 & 6 \end{vmatrix} + 0\begin{vmatrix} 1 & -4 \\ -3 & 2 \end{vmatrix} = 2(6-6) = 0$$

2. Sarrus szabály

$$\left|\begin{matrix} 1 & -4 & -2 \\ -3 & 2 & 6 \\ 0 & -2 & 0 \end{matrix}\right| \begin{matrix} 1 & -4 \\ -3 & 2 \\ 0 & -2 \end{matrix} \rightarrow \cancel{+1\cdot 2\cdot 0} + \cancel{(-4)6\cdot 0} + (-2)(-3)(-2)$$

$$\cancel{-0\cdot 2(-2)} - (-2)6\cdot 1 - \cancel{0(-3)(-4)}$$

$$= -12 + 12 = 0$$

3. Elemi átalakítások:

$$\begin{vmatrix} 1 & -4 & -2 \\ -3 & 2 & 6 \\ 0 & -2 & 0 \end{vmatrix} = -2\begin{vmatrix} 1 & -4 & -2 \\ -3 & 2 & 6 \\ 0 & 1 & 0 \end{vmatrix} = +2\begin{vmatrix} 1 & -4 & -2 \\ 0 & 1 & 0 \\ -3 & 2 & 6 \end{vmatrix}_{+3S_1} = +2\begin{vmatrix} 1 & -4 & -2 \\ 0 & 1 & 0 \\ 0 & -10 & 0 \end{vmatrix}$$

Utolsó 2 sor lin összefüggő!

$$\Downarrow$$

Egy síkban vannak

2. feladat

det $\underline{\underline{A}}$:
$$\left|\begin{array}{ccc|cc} 4 & 9 & 2 & 4 & 9 \\ 3 & 5 & 7 & 3 & 5 \\ 8 & 1 & 6 & 8 & 1 \end{array}\right| \rightarrow 4\cdot5\cdot6+9\cdot7\cdot8+2\cdot3\cdot1$$
$$-8\cdot5\cdot2-1\cdot7\cdot4-6\cdot3\cdot9$$
$$=120+504+6-80-28-162$$
$$=360$$

$\det \underline{\underline{B}} = -56$

$\det \underline{\underline{C}} = -717$

$\det \underline{\underline{D}} = -32$

3. feladat

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \qquad \text{rg}\,\underline{\underline{A}} = ?$$

Tétel (Legnagyobb el nem tűnő aldet rendje)

$\underline{\underline{A}}_1 = [1]$ $\qquad \det \underline{\underline{A}}_1 = 1 \neq 0$

$\underline{\underline{A}}_2 = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ $\qquad \det \underline{\underline{A}}_2 = 1\cdot2-3\cdot2 = -4 \neq 0$

$\underline{\underline{A}}_3 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ $\qquad \det \underline{\underline{A}}_3 = 1\cdot2\cdot3+2\cdot1\cdot2+3\cdot3\cdot1$
$$-2\cdot2\cdot3-1\cdot1\cdot1-3\cdot3\cdot2$$
$$=-12\neq0$$

$$\Downarrow$$

$$\text{rg}\,\underline{\underline{A}} = 3$$

$$\underline{\underline{B}} = \begin{bmatrix} 1 & 3 & -2 \\ -2 & -6 & 4 \\ -1 & -3 & 2 \end{bmatrix} \qquad rg\,\underline{\underline{B}} = ?$$

Ránézésre: $(1;3;-2)$, $(-2;6;4)$ és $(-1;-3;2)$ vektorok kollineárisak $\Rightarrow$ $rg\,\underline{\underline{B}} = 1$

Elemi átalakításokkal:

$$rg B = rg \begin{bmatrix} 1 & 3 & -2 \\ -2 & 6 & 4 \\ -1 & -3 & 2 \end{bmatrix} = rg \begin{bmatrix} 1 & 3 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 1$$

$$\underline{\underline{C}} = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 4 & 5 & 2 \\ -1 & 1 & -1 & 0 \end{bmatrix} \qquad rg\,\underline{\underline{C}} = ?$$

$\underline{\underline{C}} \in \mathcal{M}_{3\times4}$

$rg\,\underline{\underline{C}} \leq \min\{3;4\} = 3$

$$rg\,\underline{\underline{C}} = rg \begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 4 & 5 & 2 \\ -1 & 1 & -1 & 0 \end{bmatrix} = rg \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 2 & 1 & -4 \\ 0 & 2 & 1 & 3 \end{bmatrix} = rg \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 2 & 1 & -4 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

$$= 3$$

$$\underline{\underline{D}} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix} \qquad rg\,\underline{\underline{D}} = ?$$

$$rg\,\underline{\underline{D}} = rg \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix} = rg \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \\ 4 & 4 & 4 & 4 & 4 \\ 3 & 3 & 3 & 3 & 3 \end{bmatrix} = rg \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} =$$

$$= rg \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = 2$$

4. feladat

$$\underline{\underline{A}} = \begin{bmatrix} 1 & -1 & -3 & 3 \\ x & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 1 \end{bmatrix} \qquad rg\,\underline{\underline{A}}(x) = ?$$

$$rg\,\underline{\underline{A}} = rg \begin{bmatrix} 1 & -1 & -3 & 3 \\ x & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 1 \end{bmatrix}\begin{array}{l} -S_3 \\ \\ \\ -2S_3 \end{array} = rg \begin{array}{c} -2O_2 \\ \begin{bmatrix} 0 & -8 & -20 & 0 \\ x & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 0 & -12 & -30 & -5 \end{bmatrix} \end{array} = rg \begin{array}{c} -2O_3 \\ \begin{bmatrix} 0 & -8 & -4 & 0 \\ x & 4 & 2 & 1 \\ 1 & 7 & 3 & 3 \\ 0 & -12 & -6 & -5 \end{bmatrix} \end{array} =$$

$$= rg \begin{array}{c} -O_4 \\ \begin{bmatrix} 0 & 0 & -4 & 0 \\ x & 0 & 2 & 1 \\ 1 & 1 & 3 & 3 \\ 0 & 0 & -6 & -5 \end{bmatrix} \end{array} = rg \begin{bmatrix} 0 & 0 & -4 & 0 \\ x & 0 & 1 & 1 \\ 1 & 1 & 0 & 3 \\ 0 & 0 & -1 & -5 \end{bmatrix}\begin{array}{l} \cdot(-1/4) \\ \\ \\ \cdot(-1) \end{array} = rg \begin{array}{c} -3O_2 \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ x & 0 & 1 & 1 \\ 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{bmatrix} \end{array} =$$

$$= rg \begin{bmatrix} 0 & 0 & 1 & 0 \\ x & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}\begin{array}{l} \\ -S_1 \\ \\ -S_1 \end{array} = rg \begin{array}{c} -O_2 \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ x & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} \end{array}\begin{array}{l} \\ \\ \\ /5 \end{array} = rg \begin{bmatrix} 0 & 0 & 1 & 0 \\ x & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{array}{l} \\ -S_2 \\ \\ \\ \end{array} =$$

$$= rg \begin{bmatrix} 0 & 0 & 1 & 0 \\ x & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{cases} 3, \text{ ha } x = 0 \\ 4, \text{ ha } x \neq 0 \end{cases}$$

5. feladat

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \underline{\underline{A}}^{-1} = ?$$

$$\underline{\underline{A}} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad adj\,\underline{\underline{A}} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad det\,\underline{\underline{A}} = ad - bc$$

$$\underline{\underline{A}}^{-1} = \frac{adj\,\underline{\underline{A}}}{det\,\underline{\underline{A}}}$$

$$\left.\begin{aligned} \det \underline{\underline{A}} &= 1\cdot 4 - 3\cdot 2 = -2 \\ \operatorname{adj} \underline{\underline{A}} &= \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} \end{aligned}\right\} \underline{\underline{A}}^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

$$\underline{\underline{B}} = \begin{bmatrix} 4 & 5 \\ 2 & 6 \end{bmatrix} \qquad \underline{\underline{B}}^{-1} = ?$$

$$\ulcorner (\underline{\underline{B}} | \underline{\underline{E}}) \rightsquigarrow (\underline{\underline{E}} | \underline{\underline{B}}^{-1}) \lrcorner$$

$$\left[\begin{array}{cc|cc} 4 & 5 & 1 & 0 \\ 2 & 6 & 0 & 1 \end{array}\right] /2 \updownarrow \sim \left[\begin{array}{cc|cc} 1 & 3 & 0 & 1/2 \\ 4 & 5 & 1 & 0 \end{array}\right] -4S_1 \sim \left[\begin{array}{cc|cc} 1 & 3 & 0 & 1/2 \\ 0 & -7 & 1 & -2 \end{array}\right] /(-7)$$

$$\sim \left[\begin{array}{cc|cc} 1 & 3 & 0 & 1/2 \\ 0 & 1 & -1/7 & 2/7 \end{array}\right] \overset{-3S_2}{\sim} \left[\begin{array}{cc|cc} 1 & 0 & 3/7 & -5/14 \\ 0 & 1 & -1/7 & 2/7 \end{array}\right]$$

$$\underline{\underline{C}} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix} \qquad \underline{\underline{C}}^{-1} = ? \qquad \det \underline{\underline{C}} = -1$$

$$\underline{\underline{C}}^T = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 3 & 4 & 3 \end{bmatrix}$$

$$\operatorname{adj} \underline{\underline{C}} = \begin{bmatrix} +\begin{vmatrix} 3 & 4 \\ 4 & 3 \end{vmatrix} & -\begin{vmatrix} 3 & 4 \\ 3 & 3 \end{vmatrix} & +\begin{vmatrix} 3 & 3 \\ 3 & 4 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} \\ +\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -7 & -(-3) & +3 \\ -(-1) & 0 & -1 \\ +1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 3 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\underline{\underline{C}}^{-1} = \frac{\operatorname{adj} \underline{\underline{C}}}{\det \underline{\underline{C}}} = \begin{bmatrix} -7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$\underline{\underline{D}} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \qquad \underline{\underline{D}}^{-1} = ?$

Gauss - Jordan:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -3S_1 \\ -2S_1 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -4 & -8 & -3 & 1 & 0 \\ 0 & -3 & -3 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} \\ /(-4) \\ \\ \end{array} \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -3/4 & -1/4 & 0 \\ 0 & -3 & -3 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} -2S_2 \\ \\ +3S_2 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1/2 & 1/2 & 0 \\ 0 & 1 & 2 & 3/4 & -1/4 & 0 \\ 0 & 0 & 3 & 1/4 & -3/4 & 1 \end{array}\right] \begin{array}{l} \\ \\ /3 \end{array} \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1/2 & 1/2 & 0 \\ 0 & 1 & 2 & 3/4 & -1/4 & 0 \\ 0 & 0 & 1 & 1/12 & -1/4 & 1/3 \end{array}\right] \begin{array}{l} +S_3 \\ -2S_3 \\ \\ \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5/12 & 1/4 & 1/3 \\ 0 & 1 & 0 & 7/12 & 1/4 & -2/3 \\ 0 & 0 & 1 & 1/12 & -1/4 & 1/3 \end{array}\right]$$

6. feladat

$$\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} = (k-1)^2 - 4 \Rightarrow k-1 = \pm 4$$

$$k = \begin{cases} 3 \\ -1 \end{cases} \Rightarrow \text{ekkor nem invertálható}$$

7. feladat

$$\begin{bmatrix} x & 5 \\ -3 & z \end{bmatrix} \begin{bmatrix} 7 & y \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} -14 & 8 \\ \omega & 3 \end{bmatrix}$$

$$\begin{cases} 7x = -14 & \Rightarrow x = -2 \\ xy + 30 = 8 & \Rightarrow xy = -22 = y = 11 \\ -21 = \omega & \Rightarrow \omega = -21 \\ -3y + 6z = 3 & \Rightarrow z = \frac{1}{6}(3+3y) = \frac{1}{6}(3+33) = 6 \end{cases}$$

8. feladat

$$\underline{\underline{A}}=\begin{bmatrix}4&3\\1&1\end{bmatrix}\quad \underline{\underline{B}}=\begin{bmatrix}1&2\\3&4\end{bmatrix}\quad \underline{\underline{C}}=\begin{bmatrix}1&-1\\2&-3\end{bmatrix}\qquad \underline{\underline{A}}\,\underline{\underline{X}}+\underline{\underline{C}}=2\underline{\underline{B}}\,\underline{\underline{C}}\,\underline{\underline{X}}$$

$$\underline{\underline{A}}\,\underline{\underline{X}}-2\underline{\underline{B}}\,\underline{\underline{C}}\,\underline{\underline{X}}=-\underline{\underline{C}}$$

$$(\underline{\underline{A}}-2\underline{\underline{B}}\,\underline{\underline{C}})\underline{\underline{X}}=-\underline{\underline{C}}$$

$$\underline{\underline{X}}=-(\underline{\underline{A}}-2\underline{\underline{B}}\,\underline{\underline{C}})^{-1}\underline{\underline{C}}$$

$$\underline{\underline{A}}-2\underline{\underline{B}}\,\underline{\underline{C}}=\begin{bmatrix}4&3\\1&1\end{bmatrix}-2\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}1&-1\\2&-3\end{bmatrix}=\begin{bmatrix}4&3\\1&1\end{bmatrix}-2\begin{bmatrix}5&-7\\11&-15\end{bmatrix}=\begin{bmatrix}-6&17\\-21&31\end{bmatrix}$$

$$\left.\begin{aligned}&\hookrightarrow \text{det}: 171\\ &\text{adj}: \begin{bmatrix}31&-17\\21&-6\end{bmatrix}\end{aligned}\right\}\ (\underline{\underline{A}}-2\underline{\underline{B}}\,\underline{\underline{C}})^{-1}=\frac{1}{171}\begin{bmatrix}31&-17\\21&-6\end{bmatrix}$$

$$\underline{\underline{X}}=-\frac{1}{171}\begin{bmatrix}31&-17\\21&-6\end{bmatrix}\begin{bmatrix}1&-1\\21&-6\end{bmatrix}=\frac{1}{171}\begin{bmatrix}3&-20\\-9&3\end{bmatrix}$$

9. feladat

$$\text{rg}\begin{array}{c}\begin{bmatrix}1&2i&1+2i\\3&i&3-i\\4i&-3&-1+4i\end{bmatrix}\end{array}\begin{array}{l}\\-3S_1\\-4iS_1\end{array}=\text{rg}\begin{bmatrix}1&2i&1+2i\\0&-5i&-7i\\0&+5&+7\end{bmatrix}\begin{array}{l}\\ \\+iS_2\end{array}=$$

$$\text{rg}\begin{bmatrix}1&2i&1+2i\\0&5i&-7i\\0&0&0\end{bmatrix}=2$$