Matematika G1 - GY3 Sándor Tibor

1. feladat

a) $\overline{\left(\frac{2-i}{e^{i\cdot\pi/3}}\right)}$ nevező: $r=1$ és $\varphi=\pi/3=60°$

$e^{i\pi/3}=1\cdot(\cos 60°+i\sin 60°)=\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$$\overline{\left(\frac{2-i}{1/2+\sqrt{3}/2\,i}\right)}=\overline{\left(\frac{2-i}{(1/2+\sqrt{3}/2\,i)}\cdot\frac{1/2-\sqrt{3}/2\,i}{1/2-\sqrt{3}/2\,i}\right)}=\overline{\left(\frac{1-\sqrt{3}/2+i(-\sqrt{3}-1/2)}{1/4+3/4}\right)}$$

$a^2+b^2=(a+bi)(a-bi)=z\cdot\bar{z}$

$z_1\cdot z_2=(a_1a_2-b_1b_2)+i(a_1b_2+a_2b_1)$

$$=1-\sqrt{3}/2+i\left(1/2+\sqrt{3}\right)$$

b) $\overline{\frac{5+i}{3-2i}\cdot 3(\cos 45°+i\sin 45°)\cdot 2(\cos 270°+i\sin 270°)e^{i5\pi/12}}$

$\hookrightarrow \frac{5+i}{3-2i}=\frac{5+i}{3-2i}\cdot\frac{3+2i}{3+2i}=\frac{15-2+i(10+3)}{3^2+2^2}=\frac{13+13i}{13}=1+i$

$\hookrightarrow \overline{3\ldots}=\overline{3e^{i\pi/4}\cdot 2e^{i3\pi/2}\cdot e^{i5\pi/12}}=\overline{6\cdot e^{i\frac{13\pi}{6}}}=\overline{6e^{i\pi/6}}=6e^{i-\pi/6}$

$(1+i)\cdot 6e^{i-\pi/6}=\sqrt{2}e^{i\pi/4}\cdot 6e^{i-\pi/6}=6\sqrt{2}e^{i\pi/12}$

$=6\sqrt{2}\left(\cos\pi/12+i\sin\pi/12\right)$

$=6\sqrt{2}\left(\frac{\sqrt{2}(\sqrt{3}+1)}{4}+i\left(\frac{\sqrt{2}(\sqrt{3}-1)}{4}\right)\right)$

$=3(\sqrt{3}+1)+i\,3(\sqrt{3}-1)$

c) $\dfrac{5e^{i\frac{7\pi}{13}}}{4(\cos 135^\circ + i\sin 135^\circ)} \cdot \overline{\left(\dfrac{1}{2i}\right)} \cdot (2\sqrt{3}+2i)$

$\dfrac{5e^{i\,7\pi/13}}{4e^{i\,3\pi/4}} = \dfrac{5}{4}e^{i\frac{-11\pi}{52}}$ | $\overline{\left(\dfrac{-i}{2}\right)} = \dfrac{i}{2}$ | $4\cdot\left(\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i\right) = 4e^{i\pi/6}$

$$\frac{5}{4}e^{i\frac{-11\pi}{52}} \cdot \frac{i}{2} \cdot 4\cdot e^{i\pi/6} = \frac{5}{2}e^{i\frac{-7\pi}{156}} \cdot i = \frac{5}{2}e^{i\left(\frac{-7\pi}{156}+\frac{\pi}{2}\right)} = \underline{\underline{\frac{5}{2}e^{i\frac{71\pi}{156}}}}$$

$i = e^{i\pi/2}$

2. feladat

a) $(i-1)^{16} = \left[\sqrt{2}\left(\frac{-1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)\right]^{16} = \left[\sqrt{2}e^{i\,3\pi/4}\right]^{16} = 2^8 e^{i\frac{48\pi}{4}}$

$= 256\,e^{i\,12\pi} = 256\,e^{i\cdot 0} = \underline{\underline{256}}$

b) $(3+5i)^4 \cdot (21-35i)^5 \cdot \left(\dfrac{1+i}{1-i}\right)^4$

$(3+5i)^4 \cdot 7^5 \cdot (3-5i)^5 \cdot \left(\dfrac{(1+i)(1+i)}{(1-i)(1+i)}\right)^4 =$

$7^5\left[(3+5i)(3-5i)\right]^4 (3-5i)\left(\dfrac{1-1+2i}{1+1}\right) =$

$7^5(9+25)^4(3-5i)\,i =$

$\underline{\underline{7^5 \cdot 34^4(3i+5)}}$

3. feladat

a, $\sqrt[3]{-8} = \sqrt[3]{8(1+0i)} = \sqrt[3]{8e^{+i\pi}} = \sqrt[3]{8} \cdot \sqrt[3]{e^{+i\pi}} =$

$$= \sqrt[3]{8}\, e^{i\frac{+\pi+2k\pi}{3}} = \begin{cases} \sqrt[3]{8}\, e^{i\pi/3} & = 1+\sqrt{3}i \\ \sqrt[3]{8}\, e^{i\pi} & = -2 \\ \sqrt[3]{8}\, e^{i5\pi/3} & = 1-\sqrt{3}i \end{cases}$$

$k = \{0;1;2\}$

b, $\sqrt[4]{1} = \sqrt[4]{1+0i} = \sqrt[4]{e^{0i}} = e^{i\frac{2k\pi}{4}} \to \{1; i; -1; -i\}$

c, $\sqrt{3+4i} \Rightarrow$ nehéz átírni $\to$ trükk

$3+4i = u^2 = (x+iy)^2 = x^2-y^2+i\,2xy$

Re: $3 = x^2-y^2$

Im: $4 = 2xy \to y = 2/x$

$3 = x^2 - \left(\frac{2}{x^2}\right)$

$0 = x^4 - 3x^2 - 4 = (x^2-4)(x^2+1)$ $\to x_{12}^2 = 4$ ✓ $\to x_{34}^2 = -1$ ↯

$x_1 = 2 \quad y_1 = 1 \to z_1 = 2+i$

$x_2 = -2 \quad y_2 = -1 \to z_2 = -2-i$

4. feladat

a, $z^4 - 81i = 0 \to z^4 = 81i \to z = \sqrt[4]{81i} = 3\sqrt[4]{i}$

$= 3\sqrt[4]{e^{i\pi/2}} = 3 \cdot e^{i\frac{\pi/2+2k\pi}{4}} \qquad k = \{0;1;2;3\}$

b) $z^2-6z+13=0$

$$z_{12}=\frac{6\pm\sqrt{36-4\cdot13}}{2}=3\pm\frac{\sqrt{-16}}{2}=\underline{3\pm2i}$$

5. feladat

$$\begin{cases}z_1+2z_2=1+i & \text{(I)}\\ 3z_1+iz_2=2-3i & \text{(II)}\end{cases}$$

$$\left.\begin{aligned}&(1)\quad a_1+2a_2=1\\&(2)\quad b_1+2b_2=1\\&(3)\quad 3a_1-b_2=2\\&(4)\quad 3b_1+a_2=-3\end{aligned}\right\}\begin{aligned}&Re\{I\}\\&Im\{I\}\\&Re\{II\}\\&Im\{II\}\end{aligned}$$

$(1)\to a_1=1-2a_2=1-2(-18+18a_1)=1+36-36a_1=37-36a_1$

$(4)\to a_2=-3-3b_1=-3-3(5-6a_1)=-3-15+18a_1=-18+18a_1$

$(2)\to b_1=1-2b_2=1-2(3a_1-2)=1-6a_1+4=5-6a_1$

$(3)\to b_2=3a_1-2$

$a_1=37-36a_1$

$a_1=1$

$b_2=1$

$b_1=-1$

$a_2=0$

$z_1=1-i$

$z_2=i$

Alternatív MO:

$$\begin{cases}z_1+2z_2=1+i\\3z_1+iz_2=2-3i\end{cases}\quad -\frac{1}{3}(II)$$

$$\begin{cases}(2-i/3)z_2=1/3+2i\\3z_1+iz_2=2-3i\end{cases}\quad :\left(2-\frac{i}{3}\right)$$

$$\begin{cases}z_2=\dfrac{1/3+2i}{2-i/3}=i\\3z_1+iz_2=2-3i\end{cases}\quad -i\,(I)$$

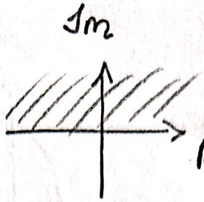
$$\begin{cases}z_2=i\\3z_1=3-3i\end{cases}\quad :3$$

$$\begin{cases}z_2=i\\z_1=1-i\end{cases}$$

3. MO: $z_2=\dfrac{1+i-z_1}{2}\ldots$

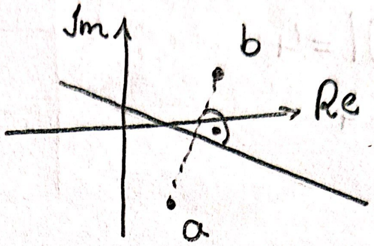
6. feladat

a, $\operatorname{Im}\{z\} > 0$



b, $|z-a| = |z-b|$

$a; b \in \mathbb{C}$



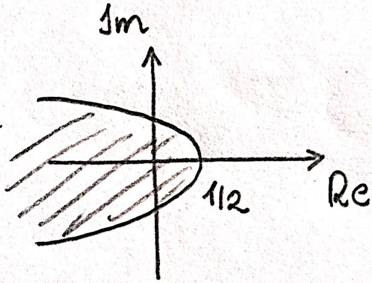
c, $|z| < 1 - \operatorname{Re}\{z\}$

$\sqrt{a^2+b^2} < 1-a$

$\cancel{a^2}+b^2 < 1-2a+\cancel{a^2}$

$2a < 1-b^2$

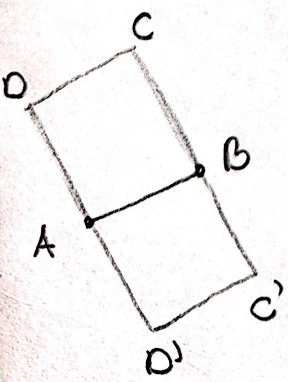
$a < \frac{1}{2} - \frac{b^2}{2}$



7. feladat

$z_1 = 3+2i$

$z_2 = 5+4i$



$\overrightarrow{AB} = z_2 - z_1 = 2+2i$

$\overrightarrow{AD} = i\overrightarrow{AB} = -2+2i$

$D = z_1 + \overrightarrow{AD} = 1+4i$

$C = z_2 + \overrightarrow{AD} = 3+6i$

$\overrightarrow{AD}' = -i\overrightarrow{AB} = 2-2i$

$D' = z_1 + \overrightarrow{AD}' = 5$

$C' = z_2 + \overrightarrow{AD}' = 7+2i$

8. feladat

$(-2;1) \Longleftrightarrow -2+i$

$r=4$

$|z-(-2+i)|=4$

$|z+2-i|=4$