Matematika G1 - GY2 · Sándor Tibor

1. feladat

$e_1: 3x - 4y - 10 = 0 \qquad \underline{n}_1 = (3; -4)$

$e_2: 6x - 8y + 5 = 0 \qquad \underline{n}_2 = (6; -8)$

$\underline{n}_1 = \lambda \underline{n}_2 \rightarrow e_1$ és e_2 párhuzamos

Tetszőleges pont: $x_0 = 0 \rightarrow e_1: 3\cdot 0 - 4y - 10 = 0 \rightarrow y_0 = \frac{-5}{2}$

$P_0 = (x_0; y_0) = (0; 5/2)$

e_2 Hesse normálalak: $e_2: \dfrac{6x-8y+5}{\sqrt{6^2+8^2}} = \dfrac{6x-8y+5}{10} = 0$

pont és egyenes távolsága:

$$\underline{\underline{d}} = \left| \frac{Ax_0 + By_0 + C}{\sqrt{A^2+B^2}} \right| = \left| \frac{6\cdot 0 - 8\left(-5/2\right) + 5}{10} \right| = \left| \frac{+25}{10} \right| = \underline{\underline{\frac{5}{2}}}$$

2. feladat

$P(-2; 5; 6)$

$Q(7; -1; 3)$

$\vec{PQ} = Q - P = (9; -6; -3) = \underline{v}$

$\underline{r}_0 = P$

$$\underline{\underline{\underline{r} = \underline{r}_0 + t\underline{v} = \begin{bmatrix} -2 \\ 5 \\ 6 \end{bmatrix} + t \begin{bmatrix} 9 \\ -6 \\ -3 \end{bmatrix}}}$$

$x(t) = -2 + 9t$

$y(t) = 5 - 6t$

$z(t) = 6 - 3t$

bármelyik alak jó

3. feladat

$$\frac{x+2}{2} = \frac{y}{-3} = \frac{z-1}{4}$$

$\underline{v}_1 = (2;-3;4)$

$\underline{r}_1 = (-2;0;1)$

$x(t) = -2+2t_1$
$y(t) = 0-3t_1$
$z(t) = 1+4t_1$

$$\frac{x-3}{a} = \frac{y-1}{4} = \frac{z-7}{2}$$

$\underline{v}_2 = (a;4;2)$

$\underline{r}_2 = (3;1;7)$

$x(t) = 3+at_2$
$y(t) = 1+4t_2$
$z(t) = 7+2t_2$

van közös pont

$x:\quad -2+2t_1 = 3+at_2$
$y:\quad 0-3t_1 = 1+4t_2$
$z:\quad 1+4t_1 = 7+2t_2$

$$y:\quad t_1 = \frac{1+4t_2}{-3}$$

$$z:\quad 1+4\cdot\frac{1+4t_2}{-3} = 7+2t_2$$

$$-3+4+16t_2 = -21-6t_2$$
$$1+16t_2 = -21-6t_2$$
$$22t_2 = -22$$
$$t_2 = -1$$

$$t_1 = \frac{1+4(-1)}{-3} = \frac{1-4}{-3} = 1$$

$x:\quad -2+2(1) = 3+a(-1)$

$$0 = 3-a$$

$$\underline{\underline{a = 3}}$$

4. feladat

$$\underline{p}_1 = \underbrace{\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}}_{\underline{r}_1} + t_1 \underbrace{\begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}}_{\underline{v}_1} \qquad \underline{p}_2 = \underbrace{\begin{bmatrix} 7 \\ 1 \\ 3 \end{bmatrix}}_{\underline{r}_2} + t_2 \underbrace{\begin{bmatrix} 6 \\ 8 \\ 4 \end{bmatrix}}_{\underline{v}_2}$$

$\underline{v}_1$ és $\underline{v}_2$ párhuzamos, legyen $P_1 = \underline{r}_1$ a fixpont

$$d = \left| \frac{\underline{v}_2 \times (\underline{r}_2 - \underline{r}_1)}{|\underline{v}_2|} \right| \qquad \underline{r}_2 - \underline{r}_1 = \begin{bmatrix} 7-2 \\ 1-(-1) \\ 3-0 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 8 \\ 4 \end{bmatrix} \times \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 16 \\ 2 \\ -28 \end{bmatrix} \qquad d = \frac{\sqrt{16^2+2^2+28^2}}{\sqrt{6^2+8^2+4^2}} = \frac{\sqrt{1044}}{\sqrt{116}} = 3$$

5. feladat

$$\underline{p}_1 = \underbrace{\begin{bmatrix} -7 \\ 4 \\ 4 \end{bmatrix}}_{\underline{r}_1} + t_1 \underbrace{\begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}}_{\underline{v}_1} \qquad \underline{p}_2 = \underbrace{\begin{bmatrix} 1 \\ -8 \\ -12 \end{bmatrix}}_{\underline{r}_2} + t_2 \underbrace{\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}}_{\underline{v}_2}$$

$$d = \left| (\underline{r}_2 - \underline{r}_1) \cdot \frac{(\underline{v}_1 \times \underline{v}_2)}{|\underline{v}_1 \times \underline{v}_2|} \right| = |(\underline{r}_2 - \underline{r}_1) \times \hat{n}_T|$$

$$\underline{v}_1 \times \underline{v}_2 = \begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 6 \\ 8 \end{bmatrix} \qquad |\underline{v}_1 \times \underline{v}_2| = \sqrt{4^2+6^2+8^2} = \sqrt{116}$$

$$\hat{n}_T = \frac{1}{\sqrt{116}} \begin{bmatrix} -4 \\ 6 \\ 8 \end{bmatrix}$$

$(\approx 21{,}54)$

$$\underline{r}_2 - \underline{r}_1 = \begin{bmatrix} 8 \\ -12 \\ -16 \end{bmatrix} \qquad d = \left| \frac{1}{\sqrt{116}} \left(8(-4) + (-12)6 + (-16)8 \right) \right| = 4\sqrt{29}$$

Normáltranszverzális -> távolság minimum

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2} = \sqrt{(-7+3t-1-t)^2 + \dots}$$

$$= \dots \sqrt{36t^2 + 464} \to t = 0\text{-nál minimális}$$

$$\underline{r}_n = (-7;4;4) \to \underline{p}_n = \underline{r}_n + t\hat{n}_T$$

6. feladat

$P(0;-1;2)$

$Q(2;-1;1)$

$R(4;3;-2)$

$$\vec{PQ} \times \vec{PR} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 8 \end{bmatrix}$$

$\neq \underline{0} \to$ nem esnek egy egyenesbe

Sík: $\underline{n} = \vec{PQ} \times \vec{PR} = (4;4;8) = (A;B;C)$

$\underline{r_0} = P = (0;-1;2)$

$\underline{n} \cdot \underline{r_0} = 4 \cdot 0 + 4(-1) + 8 \cdot 2 = +12 = -D$

$\underline{\underline{4x + 4y + 8z - 12 = 0}}$

7. feladat

$e: \dfrac{x-1}{2} = \dfrac{y-2}{2} = \dfrac{z}{a} \longrightarrow \underline{v} = (2;2;a)$

$\underline{n} = (1;3;-2a)$

$s: x + 3y - 2az = 0$

$\underline{v} \cdot \underline{n} = 2 \cdot 1 + 2 \cdot 3 + a(-2a) = 0$ (merőlegesek)

$8 - 2a^2 = 0$

$\underline{\underline{a = \pm 2}}$

8. feladat

$e: x - 1 = \dfrac{y+1}{-2} = \dfrac{z}{6}$

$x(t) = 1 + 1t = 1 + t$

$y(t) = -1 - 2t = -1 - 2t$

$z(t) = 0 + 6t = 6t$

$s: 2x + 3y + z - 1 = 0$

$\hookrightarrow 2(1+t) + 3(-1-2t) + 6t - 1 = 0 \to 2 + 2t - 3 - 6t + 6t - 1 = 0$

$t = 1$

$x(1) = 2$

$y(1) = -3$

$z(1) = 6$

$P(2;-3;6) \longleftarrow$

9. feladat

$s_1: \; x - 2y + 3z - 4 = 0 \qquad \underline{n}_1 \, (1; -2; 3)$

$s_2: \; 3x + 2y - 5z - 4 = 0 \qquad \underline{n}_2 \, (3; 2; -5)$

$$\underline{v} = \underline{n}_1 \times \underline{n}_2 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 4 \\ 14 \\ 8 \end{bmatrix}$$

Fixpont: $z = 0$ (bármi mást is fixálhatnánk)

$$\left.\begin{array}{ll} (1) & x - 2y = 4 \\ (2) & 3x + 2y = 4 \end{array}\right\} \qquad \left.\begin{array}{ll} (1)+(2) & 4x = 8 \\ & x = 2 \\ (1) & 2 - 2y = 4 \\ & y = 1 \end{array}\right\} \qquad \begin{array}{l} \underline{r}_0 (2; 1; 0) \\ \\ \underline{r} = \underline{r}_0 + t\underline{v} \end{array}$$

10. feladat

$$\left.\begin{array}{ll} (1) & 2x + y - z - 2 = 0 \\ (2) & x - 3y + z + 1 = 0 \\ (3) & x + y + z - 3 = 0 \end{array}\right\}$$

$(1)-(2)-(3) \rightarrow 3y - 3z = 0$

$y = z$

$(1) \quad 2x = 2 \rightarrow x = 1$

$(2) \quad 1 - 3y + y + 1 = 0 \rightarrow y = z = 1$

$x + y + 2z = 0$

$$\left.\begin{array}{l} \underline{r}_0 = (1; 1; 1) \\ \underline{n} = (1; 1; 2) \\ \underline{r}_0 \cdot \underline{n} = 4 \end{array}\right\} \quad x + y + 2z - 4 = 0$$